



## **Theoretical Ship Wave Pattern Resistance Evaluation Using Kochin Wave Amplitude Function**

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### ***Abstract:***

*Ship wave pattern resistance evaluation has been an important event in the optimization process of variety of hull forms ranging from small crafts to fast displacement hull forms including multihull vessels. Several experimental methods for wave pattern analysis like Hogben Matrix Element Method (MEM), Longitudinal & transverse wave cuts involving Landweber Fourier Transform (LFT) and Matrix solution method, theoretical methods like Michell's thin ship theory, Rankine source-sink and Kochin-Fourier method exist today with their merits and limitations. Ship wave pattern resistance is mostly a derived quantity from experiments either as a subset of residuary resistance or when total viscous component is known. In general applying Froude's hypothesis, the residuary resistance coefficient is not subjected to scaling effects from model to prototype and evaluation of total viscous component for surface ships by model experiments is seldom done. Optimization of hull forms through wave pattern studies is a powerful tool and in-house potential flow solvers/experimental evaluation techniques for free surface flows have been developed in many hydrodynamic test facilities. The present paper focuses on ship wave pattern evaluation using Michell's theory (using Kochin wave amplitude function) and its comparison with Shipflow<sup>®</sup> & Longitudinal wave cut method (using LFT) on bench mark test model R/V Athena. Matlab<sup>®</sup> code development and ship wave pattern resistance evaluation sensitivity for the method chosen is also a part of present discussion.*

## **1.Introduction**

The demand for high performance ships that develops better speed at lower power demand and better ride comfort in a seaway has increased in the recent past. To develop optimal hull forms for such vessels, thorough investigation into resistance and other hydrodynamic characteristics of the designs have to be performed. Wave making resistance is an important component of total hydrodynamic drag for surface ships and deals with the energy required to maintain the wave system. Several wave pattern analysis methods, both analytical [4, 19] and experimental [1, 20], have been developed in order to assess the wave making resistance qualitatively and quantitatively and also as a tool for the hull form optimization process.

Determination of the resistance to the motion of a body in fluid (liquids) is a fundamental problem in hydrodynamics. This problem, difficult enough for arbitrary shaped bodies which are completely immersed, is even more complex for surface ships which are only partially immersed (multi-phase flow phenomenon). Hydrodynamics for surface ships comprises of free surface effects and viscous effects. The physical modelling of free surface effects is complex. The wave making resistance (mainly residuary resistance) is conventionally determined through hydrodynamic model tests in a towing tank using ITTC method [7].

Perform analytical and numerical analysis in tandem with experiments for estimating the wave making resistance of ships will surely optimise the model testing matrix. A thorough validation of these non-experimental methods using published bench mark data is very important in further application of such methods [1]. The present paper makes an attempt to determine the wave making resistance using analytical method – Kochin amplitude wave function using Michell's thin ship theory and its validation with existing published results on R/V Athena [26] & commercially available software Shipflow<sup>®</sup>

## **2.Techniques/Methods For Estimation Of Wave Making Resistance**

Wave making resistance has been obtained using analytical/numerical and experimental techniques [1] as listed in table below.

S.No	Techniques	Methods
1	Experimental (wave cuts)	-Fourier Transform -Matrix solution -Spectral analysis
2	Analytical	-Michell's thin ship theory -Rankine source distribution
3	Numerical	-CFD methods

Table 1

The wave making resistance estimation through experimentation is normally conducted in a towing tank. Wave probes connected to a data acquisition system are used to measure longitudinal wave traces. Capacitance type wave probes are positioned on one side of the tank in transverse direction in order to conduct the multiple longitudinal cut wave pattern analysis. The wave pattern of a ship model, symmetric relative to the tank centreline, can be expressed as a summation of a number of free waves,

$$\zeta = \sum_{n=0}^{\infty} (\xi_n \cos xw_n + \eta_n \sin xw_n) \cos\left(\frac{2\pi ny}{W}\right)$$

The wave pattern analysis aims to derive the wave coefficients  $\xi_n$  and  $\eta_n$  from the wave traces taken either across the tank or along the tank by employing analysis methods. Four main analysis techniques are available.

### 2.1. Fourier Transformation

The free-wave spectrum can be determined by Fourier transformation of either longitudinal or transverse wave cut data for the steady-ship wave system in the time domain. Standard procedure is described in detail by Eggers et al.

### 2.2. Matrix Solution Method

The method uses simultaneous equations for solving the wave coefficients. The errors due to truncation of wave trace length and due to non periodic wave trace can be prevented by applying this method [1].

### 2.3. Spectral Density Method

Wave profiles at a particular probe distances is utilised for estimating wave spectrum and thus analysed to obtain the wave energy /wave making resistance.

### 3. Kochin Wave Amplitude Function Methodology

The steady surface wave pattern  $z = \zeta(x, y)$  of any moving body, as seen at a point  $(x, y)$  sufficiently far from the ship, is of the form of a sum of plane waves travelling at various angles  $\theta$  of propagation relative to the direction of motion (negative  $x$ -axis) of the body.

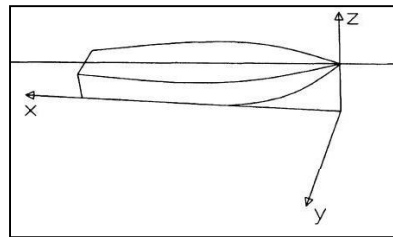


Figure 1: Coordinate system definition

$$\zeta(x,y) = \Re \int_{-\pi/2}^{+\pi/2} A(\theta) e^{-i\Omega(\theta)} d\theta$$

$$\Omega(\theta) = k(\theta)[x\cos\theta + y\sin\theta]$$

$\Omega(\theta)$  is a phase function and  $A(\theta)$  is the amplitude and  $k(\theta)$  the wave number of the wave component travelling at angle  $\theta$ . Note that the contributions to this integral from *positive* angles  $\theta$  correspond to waves being propagated to the *left* or *portside* of the body. In the present report the body and flow are assumed to have lateral symmetry, and then  $A(\theta)$  is an even function and need only be computed for  $\theta \geq 0$ . Once  $A(\theta)$  and  $k(\theta)$  are specified, we can use this to determine the actual wave pattern. At any given speed  $U$ , the amplitude function  $A(\theta)$  is a property only of the body's hull geometry, whereas  $k(\theta)$  is fully determined from the dispersion relation for plane waves. In infinite water depth, which we shall assume in the present report, we have simply

$$k(\theta) = k_0 \sec^2 \theta$$

Where  $k_0 = g/U^2 = k(0)$  is the wave number of pure transverse waves at  $\theta = 0$ . The complex amplitude function  $A(\theta)$ , sometimes also called the free wave spectrum or Kochin function, can be computed by various means, e.g. Michell's thin-ship theory, a

complete nonlinear near-field computation or by experimental measurement . For a thin body with offsets  $y = Y(x, z)$  in water of infinite depth, Michell's theory indicates that

$$A(\theta) = -\frac{2i}{\pi} k_0^2 \sec^4 \theta \iint Y(x, z) \exp(k_0 z \sec^2 \theta + ik_0 x \sec \theta) dx dz$$

The total energy left behind in the wave field can easily be computed once the amplitude function  $A(\theta)$  is known, which leads to the famous Michell's integral for the wave resistance  $R$  for a ship, namely

$$R_{WP} = \frac{\pi}{2} \rho U^2 \int_{-\pi/2}^{+\pi/2} |A(\theta)|^2 \cos^3(\theta) d\theta$$

The thin-ship theory of Michell [1] represents the body by a centre plane source distribution proportional to its longitudinal rate of change of thickness (local beam). The only requirement for its validity is that the quantity be small. Hence the theory applies to submerged as well as surface-piercing bodies. The double integral over the body's centre plane, determining  $A(\theta)$  for each fixed angle  $\theta$  from the offsets  $Y(x, z)$ , can be reduced to a pair of separate integrals over depth and length as follows. Evaluation of  $F(x, \theta)$  for all stations  $x$  is attempted in the first phase.

$$F(x, \theta) = \int Y(x, z) \exp(k_0 z \sec^2 \theta) dz$$

The integral is in the vertical  $z$ -direction, from the lowest point of the section in its actual position with  $z < 0$ , up to the waterline  $z = 0$ . Next, evaluation of pair of integrals  $P(\theta), Q(\theta)$  along the body from bow to stern is performed.

$$P(\theta) = \int F(x, \theta) \cos(k_0 x \sec \theta) dx$$

$$Q(\theta) = \int F(x, \theta) \sin(k_0 x \sec \theta) dx$$

Amplitude function can be finally represented as

$$A(\theta) = -\frac{2i}{\pi} k_0^2 \sec^4 \theta [P(\theta) + iQ(\theta)]$$

There are a number of special features, especially the singularity as  $\theta \rightarrow \pi/2$ . In that limit (which corresponds to the diverging part of the ship wave pattern), the effective wave

number  $k_0 \sec \theta$  becomes infinite, and the  $x$  integral has a highly-oscillatory integrand. This is a well-understood problem, but it does require some care, especially in data specification near the bow and stern, which contribute most to the integrals in that limit. It is moderated, especially for submerged or semi submerged hulls, by the exponential decay factor from the  $z$  integral that damps out contributions from near  $\theta = \pi/2$  [19]. If there is no contribution from the extreme bow and stern (i.e.  $Y = 0$  and hence  $F = 0$  there), the algorithm used is simply

$$P(\theta) \approx \sum_{i=1}^{N_x-1} (\omega_i F(x_i, \theta) (\cos k_0 x_i \sec \theta) \Delta x)$$

$$w_{2i} = (3K + K \cos 2K - 2 \sin 2K) / K^3$$

$$w_{2i+1} = 4(\sin K - K \cos K) / K^3$$

where  $K = k_0 \sec \theta \Delta x$ .

$$F(x, \theta) \approx \sum_{j=0}^{N_z} (\omega_j Y(x, z_j) \exp(k_0 z_j \sec^2 \theta) \Delta z)$$

where the section is divided into  $N_z$  segments each of height  $\Delta z$ , with end-points  $z_j$ ;  $j = 0, 1, 2, \dots, N_z$ , and the weights are given by  $\omega_j$

where

$$\omega_0 = (e^K - 1 - K) / K^2$$

$$\omega_{N_z} = (e^{-K} - 1 + K) / K^2$$

$$\text{and for } j \neq 0, N_z, \quad \omega_j = (e^K + e^{-K} - 2) / K^2$$

where,  $K = k_0 \sec^2 \theta \Delta z$ .

wave making resistance coefficient can be determined as follows

$$C_W = \frac{R_{WP}}{\frac{1}{2} \rho U^2 S} = \int_{-\pi/2}^{+\pi/2} C_{WP}(\theta) d\theta$$

#### 4.Presentation Of Results

Matlab<sup>®</sup> code using Kochin amplitude wave function method was generated for the evaluation of wave making resistance and the CFD analysis was performed using Shipflow<sup>®</sup>. A bench mark ship (R/V Athena) has been selected for the application of the present code and its validation with the published experimental data as well with Shipflow<sup>®</sup>. The experiments conducted at NSWC [26] and NSTL [27] has been utilised for validation. Main particulars of R/V Athena are given in Table 2.

S.No.	Parameter	
1	$L_{BP}$	46.87 m
2	B	6.68 m
3	$\Delta$	260 tonnes
4	U	12 kn - 35 kn

Table 2

The evaluation of the code was done in a Froude number range of 0.25 to 0.7 for a fixed sinkage and trim and comparison of the wave profiles at  $y/L = 0.2$  for  $Fn$  0.257 & 0.44 is given in Fig 3 & 4.

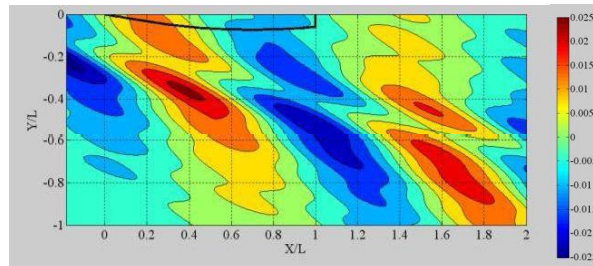


Figure 2: Wave contour for  $Fn = 0.44$

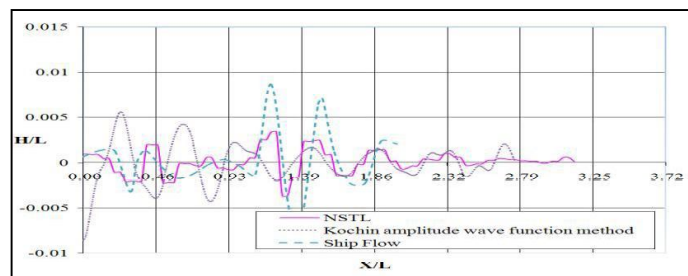


Figure 3: Wave heights at  $y/L = 0.2$  for  $Fn = 0.257$

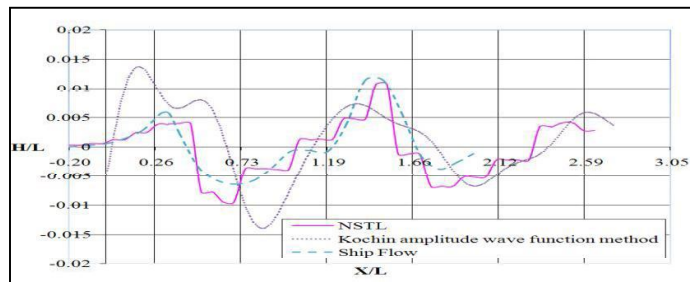


Figure 4: Wave heights at  $y/L = 0.2$  for  $Fn = 0.44$

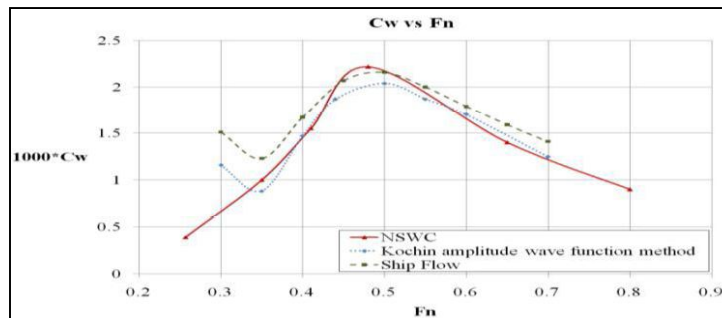


Figure 5: Coefficient of wave resistance vs Froude number

## 5. Conclusion

The present paper successfully made an attempt through Matlab code for evaluation of wave making resistance of a surface ship using Kochin amplitude wave function. Fig 3 & 4 shows a comparison of the present analytical method. Fig 5 shows a comparison of wave making resistance coefficient with respect to Froude number. The wave profile comparison for Froude number less than 0.3 shows more number of wave oscillation than with experiments, but the amplitudes after  $1.2L$  are nearly same and the same is the behaviour with the phase angles. For  $Fn > 0.3$  the wave amplitudes & phase angles have a very fair match with experiments. Shipflow results too behaved in a similar manner.

Wave contour plot in Fig 2 shows the wave pattern for the present ship at  $Fn 0.44$ , which seems to be simulating the actual physics. One can observe wave crest at bow region (red in colour) and trough at the after location (slightly blue in colour). Fig 5 plot shows the reliability of the present method in evaluating the wave making resistance.  $C_w$  plot is in close agreement with both experiments and Shipflow<sup>®</sup> results. Except at lower Froude numbers ( $Fn < 0.3$ ) where the  $C_w$  from present method as well as from Shipflow<sup>®</sup> diverged, the rest of the curve has a trend. Table 3 gives the values of wave making resistance comparison at  $Fn 0.257$  &  $0.44$ .



	<b>Fn = 0.257</b>		<b>Fn = 0.44</b>	
	R <sub>w</sub>	10 <sup>3</sup> C <sub>w</sub>	R <sub>w</sub>	10 <sup>3</sup> C <sub>w</sub>
<b>NSWC</b>	2.98	0.39	36.96	1.70
<b>Shipflow<sup>®</sup></b>	13.6	1.75	41.6	1.91
<b>Kochin wave amplitude function method</b>	12.54	1.69	40.64	1.86

*Table 3*

The mathematical code generated for the evaluation of wave making resistance have given the results very close to the experimental and the Shipflow results. Hence the code can be used for the preliminary estimation of wave making resistance and optimization the hull form. Further the code can be improved by incorporating free sinkage & trim to simulate the dynamic motion conditions and also its extension to multihulls.